Credit
Research
Center

REPRINT No. 4

Credit Screening System Selection

1976

Krannert Graduate School of Management – Purdue University

JOURNAL OF FINANCIAL AND QUANTITATIVE ANALYSIS JUNE 1976

CREDIT SCREENING SYSTEM SELECTION

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Recent financial literature has discussed how a creditor[1] should determine its investigation and extension policy. Mehta [8,9] has developed a sequential process for credit extension, and others [1,2,4,7,10,12,14] have used credit-scoring functions to develop decisions rules. Instead of discussing the use of a particular system or the development of a new system, this paper shifts the focus to selection of the best of alternative systems. Different creditors face different profit-loss ratios on loans, business volume, and prior probabilities of good and bad customers. Furthermore, since the alternative systems have different initial costs, effectiveness, and investigation costs per application, no one system is optimal for all creditors. Finally, any credit-scoring alternative declines in effectiveness over time. Measurement of the overall effectiveness of a system requires that the optimal time between updating the system be known.

The first section develops the system selecting using maximization of expected net present value as the criteria. The proposed method can incorporate changes over time, making it more flexible than the original work in the area by Edmister and Schlarbaum [3]. The second section develops the criteria for determining an updating cycle for credit-scoring systems that maximizes the expected present value, using a method analogous to equipment replacement under certainty. The third section determines the optimal updating period of a credit-scoring system. The model then considers the effect different decay rates, costs of capital, growth rates, and relative updating costs have on the optimal updating period. Given estimates of the systems performance and decay over time, the fourth section shows how to select the decay function and its parameter giving the smallest predictive variance.

I. System Selection

The basic objective of management is posited to be maximization of the market value of the common shareholder's equity. Because market value of these shares is assumed to be determined by investors who discount the firm's expected net cash flows at a constant rate over time, it follows that the objective of credit analysis is to maximize the expected net present value of after tax cash flows from granting cash credit to applicants.[2] As the net after-tax cash flows determine the firm's value, all subsequent revenues and costs are figured net of taxes.

Edmister and Schlarbaum [3] developed the first model for determining the optimal system. Equation (1) presents their model for the i^{th} system where N_{gi} and N_{bi} are the respective number of good and bad customers granted credit, R and L are the average return and losses. The cost of processing N applicants is shown as S_i (N), and G is the fixed overhead.

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[*] *Georgia Institute of Technology.* *Preparation of this paper has been aided by the Credit Research Center, Purdue University. Special thanks to Robert W. Johnson for his helpful comments and guidance in preparing this paper. The views expressed and any errors are those of the author.*

[1] The methodology can be readily extended to sales credit, but the discussion has been couched in terms of cash credit for ease of exposition. Sales credit creates an additional difficulty in determining the correct profit from good customers and loss from bad customers.

[2] The firm's cost of capital could vary under various alternative credit screening systems. Different systems probably give different default rates and screening costs. Because a system has a higher default rate does not necessarily mean that it has a higher cost of capital. This greater default rate is diversified over many loans. The effect of the default rate on systematic risk that in turn affects the cost of capital is difficult to determine. Therefore, a constant cost of capital is used throughout this section.

(1) $E[PV_i] = N_{gi} R - N_{bi} L - S_i (N) - G.$

The system giving the maximum E[PV] is selected.

Their system implicitly assumes constant conditions over time. Their costs must include no initial start-up costs or periodic updating costs.[3] Because their system's performance must be constant over time, some average performance estimate must be used when a credit-scoring system's efficiency declines over tine. Nor can their approach evaluate the effect of changes in the expected number or quality of loan applicants on the system selected. The system set forth here handles these problems, since the expected values of each of the key independent variables are a function of time as well as of the system selected.

The cash flows related to credit extension are broken into four parts for better analysis. First, certain costs are unaffected by the system selected. These general costs (G) include costs from being in business such as fixed office expense, rent, utilities, supplies, and management overhead expenses.

Second, each of the i^{th} systems considered has its operational screening expenses per applicant (S_i). These costs are fixed for a given system with a known number of applicants (N) to process. When the optimal cutoff criterion for an individual applicant is considered, these costs are not included in the analysis of profitability. However, when an entire system is under evaluation, the screening costs are variable.

Third, the total expected contribution from accepted good customers equals the net contribution per loan (R_{ki}) summed over all good customers, $k=1,2,...,N_g$, granted credit under the i^{th} system for periods $j=1,2,\ldots, \omega$. The net contribution per loan is the present value of a loan's payments net of variable trans action expenses and income taxes, minus the initial principal and the net variable costs after taxes of making the loan. These returns and variable costs per loan (but not the number of good customers receiving loans) are independent of the screening system selected.

Fourth, the total expected loss from customers that default equals the net loss per loan (L_{kj}) summed over all bad customers granted credit for the specific system (N_{bij}). The net loss per loan is the sum of the principal, the net variable cost after taxes of making the loan, and the net present value of any collection expenses minus the present value of any payments received net of cash outflows for taxes and variable transaction expenses, proceeds from collateral, and the tax savings from the principal written off. The loss per loan is assumed to be independent of the screening system used.

The expected present value for a given screening system (i) can now be expressed by equation (2)

(2) $$E\left[PV_i\right] = \sum_{j=0}^{\infty} \frac{1}{(1+\rho)^j} \left(\sum_{k=1}^{N_{gij}} R_{kj} - \sum_{k=1}^{N_{bij}} L_{kj} - S_{ij}(N_j) - G_j \right)$$

where ρ is the cost of capital for the firm.

Two points need emphasizing. First, N_{gij} includes only the potentially good customers that receive credit and will be somewhat less than the total number of good customers. This is reflected in a lower total contribution value. Second, the N_{bij} term considers only bad customers that can be expected to receive credit and incur losses with the particular screening system. Obviously N_{gij} and N_{bij} will vary with the screening system. A perfect system would have N_{gij} include all potentially good customers and N_{bij} equal zero.

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[3] This poses no serious difficulty as equivalent annual costs can be used. See Johnson [6] for a discussion on E.A.C.

Selection of the optimal screening system is now quite straightforward: estimate the parameters for each alternative system, calculate the expected present values, and select the system with the largest expected present value. Alternative systems could include, for example, a purely subjective evaluation system, a credit-scoring system, a combination of the two, or a sequential decision process.

II. Development of the Optimal Updating Schedule

The optimal updating cycle for each credit-scoring system under consideration must be known to estimate the parameters in equation (2). Since it is assumed that no significant innovations in scoring techniques are introduced, each updated system is of the same quality as the original system when it was new. Solving for the optimal updating cycle is analogous to equipment replacement under certainty. The equipment is a credit-scoring system that decays over time. As the particular population changes, and also the overall standard and patterns of living change, the factors that estimate an individual credit worthiness change. For example, originally having a telephone is a positive factor in identifying credit worthiness, but over time a vast majority of applicants obtain telephones. Consequently having a telephone loses its importance in identifying credit worthiness.

The use of a certainty replacement Period in an uncertain situation needs some justification. If much innovation is expected in the field, obviously uncertainty should be considered. While shifts occur in the relative importance of variables over time, credit scoring has not changed greatly in basic technique or efficiency, nor can a significant increase in performance be anticipated in the near future. Although sudden shifts in an office's clientele could cause serious problems, a transition normally occurs relatively slowly. Any sudden shift could probably be traced to new locations or changes in promotion. Since these are management-controlled items, their likelihood can be estimated and incorporated into the model.

Any decay in a credit-scoring system must be compared to the results of using no system. Whereas firms with a very high prior probability of good customers may not experience a noticeable decay in a system's discriminatory power, other firms with low prior probabilities of good customers face what appears to be a rapid decay rate. Actually, both systems can be decaying at the same rate. To correct for this factor, the return from using no system should be subtracted from the return of the system.[4] The index of decay can then be

expressed as $\frac{\text{Current Return}}{\text{New Return}} - 1.$

To simplify equation (2), let $M_o = \sum_{k=1}^{N} g_{ij} R_{kj} - \sum_{k=1}^{N} b_{ij} L_{kj}$,

the expected yearly rate of the contribution's present value from good customers minus losses for the new system from loans made that year. Further, add and subtract the present value that would occur with no selection system, 0_0.

The S_{ij} (N_J) and G values are broken into three parts. First, G (N) is the yearly processing expense for N applicants and general overhead expense. Second, U isthe periodic updating cost for the credit-scoring system at intervals of T years. And third, I is a one-time initial expense for starting up the system. Finally, using continuous discounting for the bracketed portion, equation (3) is obtained.

[4] This assumes that the overall population of credit applicants is unaffected through complete credit extension with no screening system. For comparative purposes here, no difficulty arises. However, when considering the alternative of complete credit extension, shifts in the population could become a very important factor.

$$(3)\quad E\left[PV_i\right] = \sum_{i=0}^{\infty} e^{-\rho T i}\left[\int_0^T O_o e^{-\rho t}dt + \int_0^T (M_o - O_o) f(t) e^{-\rho t}dt - \int_0^T G(N)\, e^{-\rho t}dt - Ue^{-\rho T}\right] - I.$$

Possible growth in the number of applicants over time has been neglected in equation (3). Earlier, in equation (2), the expected number of good and bad applicants could vary over time, thus allowing for growth. In contrast with the continuous function, growth must be more closely defined to maintain integrability. Therefore, any growth must be at the same rate for both goods and bads and continuously compounded for an infinite duration. It is also assumed that the annual net inflows (M_0) and screening cost per applicant rise at the growth rate while the updating cost is constant. Equation (4) gives the new growth adjusted, expected present value formulation. Note that the growth rate must be less than the discount rate to give a finite value for the expected present value.

$$(4)\quad E\left[PV_i\right] = \sum_{i=0}^{\infty} e^{iT(g-\rho)}\left[\int_0^T O_o e^{t(g-\rho)}dt + \int_0^T (M_o - O_o) f(t) e^{t(g-\rho)}dt - \int_0^T G(N)\, e^{t(g-\rho)}dt\right] - \sum_{i=0}^{\infty} e^{-iT\rho}\left[Ue^{-\rho T}\right] - I$$

First-order difference equations are then used to eliminate the summation.

(5) $$E\left[PV_i\right] = \frac{1}{1-e^{T(q-\rho)}}\left[\int_0^T O_o e^{t(q-\rho)}dt + \int_0^T (M_o - O_o) f(t) e^{t(q-\rho)} dt - \int_0^T G(N)\, e^{t(q-\rho)} dt\right] - \frac{Ue^{-\rho T}}{1-e^{-\rho T}} - I$$

The first and third integrals can be solved directly, and letting $E(T) = \int_0^T f(t)e^{t(q-\rho)}dt$, the expected present value is determined

(6) $$E(PV_i) = \frac{O_o}{(\rho-q)} + \frac{(M_o-O_o)E(T)}{1-e^{T(q-\rho)}} - \frac{G(N)}{(\rho-q)} - \frac{Ue^{-\rho T}}{1-e^{-\rho T}} - I.$$

III. Optimal Updating Period and Parameter Effects

Noting that $\frac{dE(T)}{dT} = f(T)$, equation (6) is differentiated with respect to T and set equal to zero to determine the optimal updating cycle.

(7) $$0 = \frac{dE(PV_i)}{dT} = \frac{(M_o-O_o)\, f(T) e^{T(q-\rho)}}{(1-e^{T(q-\rho)})} + \frac{(M_o-O_o)\, e^{T(q-\rho)}\, (q-\rho)E(T)}{(1-e^{T(q-\rho)})^2} + \frac{U\rho e^{-\rho T}}{(1-e^{-T\rho})^2}$$

Multiplying both sides of equation (7) by $(1-e^{T(q-\rho)})^2\, e^{\rho T}$ simplifies the relationship. Dividing through by (M_o-O_o) expresses the updating cost as a ratio with the system's benefit which eliminates dollar amounts.

(8) $$0 = (1-e^{T(q-\rho)})\, f(T)\, e^{Tq} + E(T)(q-\rho)e^{Tq} + \frac{\rho\, U\, (1-e^{T(q-\rho)})^2}{(M_o-O_o)(1-e^{-T\rho})^2}\ .$$

The effect of the various Parameters on the optimal updating cycle can be expressed best graphically. A constant or linear decay function ($f(t) = (1-\alpha t^2)$) and a quadratic decay function ($f(t) = (1\text{-at } 2)$) are considered here for expository purposes. Four parameters -- the decay rate, the relative updating cost, the growth, and discount rate--affect the cycle's length.

Given the specification of any three of these four variables, the relationship of the fourth variable to the optimal updating period can be determined. The process also permits an examination of the sensitivity of the opt mal period to variations in the unspecified parameter. The standard reference values used in the following illustrations are cost of capital, 6 percent; growth rate, 2 percent; cost ratio, 0.10; linear decay rate, 0.02; and quadratic decay rate, 0.004.

As shown in Figure I, the optimal updating cycle is inversely related to the decay rate. Also, for a given decay rate the updating period is consider ably shorter with the quadratic decay. At the decay rates specified above, the optimal updating periods are 2.7 years and 3.0 years for linear and quadratic decay, respectively.

Figure II shows the relationship of the optimal updating period to the cost ratio. Using the ratio of the net updating cost to the difference between a new system's present value and that of no screening system, both decay functions show a longer updating period with a higher cost ratio. However, with quadratic decay, the cost ratio's effect on the optimal updating cycle is not as great. The new system's value ($M_o - 0_o$) is a function of the firm's overall volume, whereas the net updating cost is basically independent of volume: the larger the firm, the lower the updating cost ratio and the shorter the updating cycle.

Figure III shows the effect of the cost of capital on the optimal updating period. Both decay functions move upwards together. The effect of cost of capital is most pronounced at very low levels, since with a 2 percent growth rate the effective discount rate is only 1 percent. At higher levels, the effect of the cost of capital on the optimal updating period is very slight. For example, doubling the cost of capital from 5 Percent to 10 percent increases the optimal updating period only from 2.9 to 3.3 years with the quadratic decay.

The relation of the business growth rate to the optimal updating cycle is found in Figure IV. Both decay functions have a shorter updating period with greater growth rates. With a linear decay function, increasing growth from 2 percent to 4 percent decreases the updating period from 2.7 years to 1.9 years, while the same growth rate change causes a decrease in the cycle length from 3.0 years to 2.4 years with a quadratic decay.

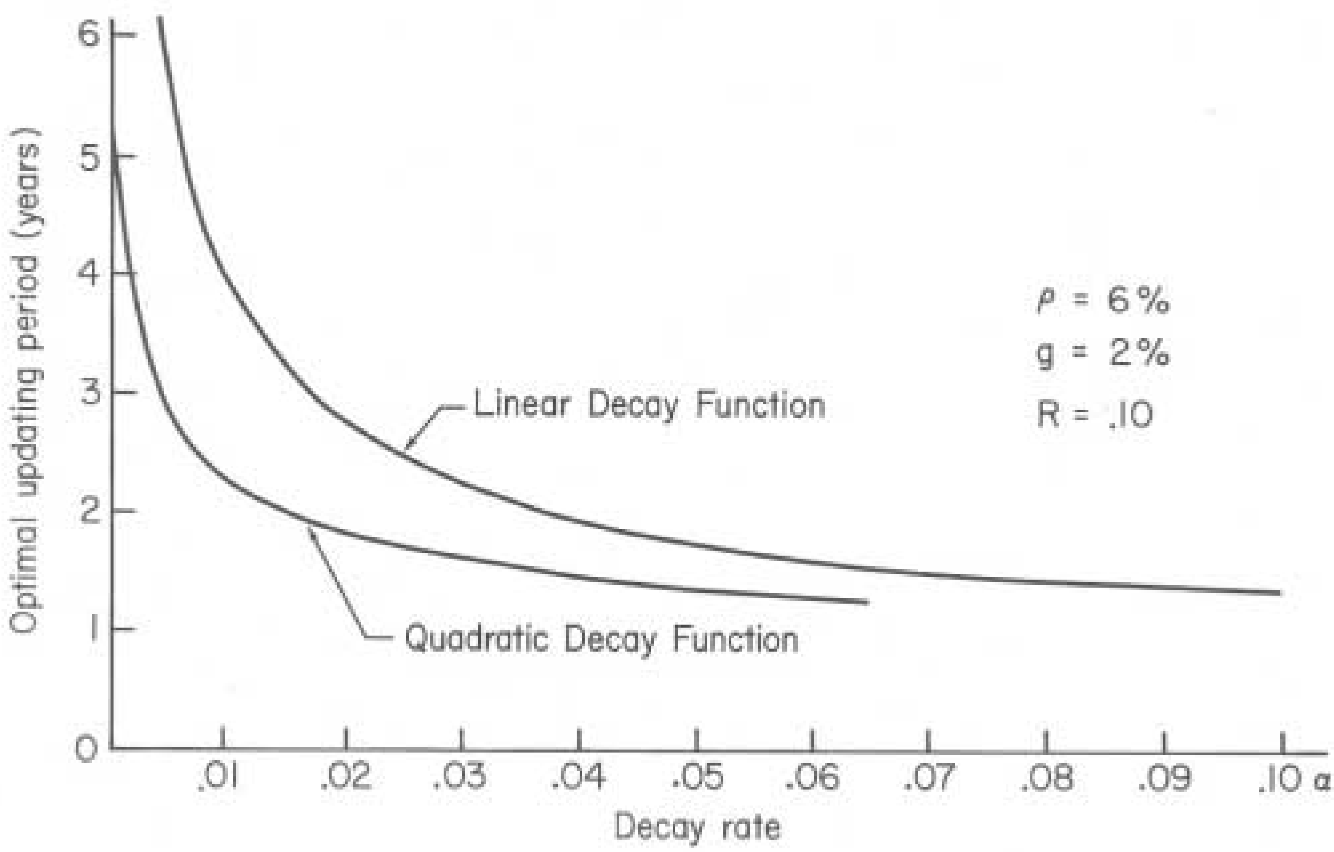

FIGURE I

DECAY RATE VS. OPTIMAL UPDATING SCHEDULE

IV. Measuring the Decay

The system developed is based upon knowing the decay function and its decay rate. To estimate these values, the credit-scoring system's performance over time must first be measured, and then the decay function having the minimum variance is selected to solve for the optimal length of the updating cycle.

The initial $(M_o - 0_o)$ must be determined before considering the decay. As noted above, M_0 is the resent value of the expected contribution after taxes before fixed expenses of a new system. It is estimated by applying the new system to a holdout sample with the same proportion of goods and bads as the actual population. The optimal cutoff score for a priori classification of goods and bads is that which minimizes the costs of misclassification. The expected returns on good loans are multiplied by the number of goods classified as good, and from this is subtracted the expected losses on bad loans times the number of bads classified as goods. Then, M_o is merely the holdout result scaled upwards by $\frac{\text{Number of actual applicants}}{\text{Number in the holdout sample.}}$

0_0 is the present value of the expected return from credit extension without any screening system. The difference between M_0 and 0_0 is the increased value of a new system.

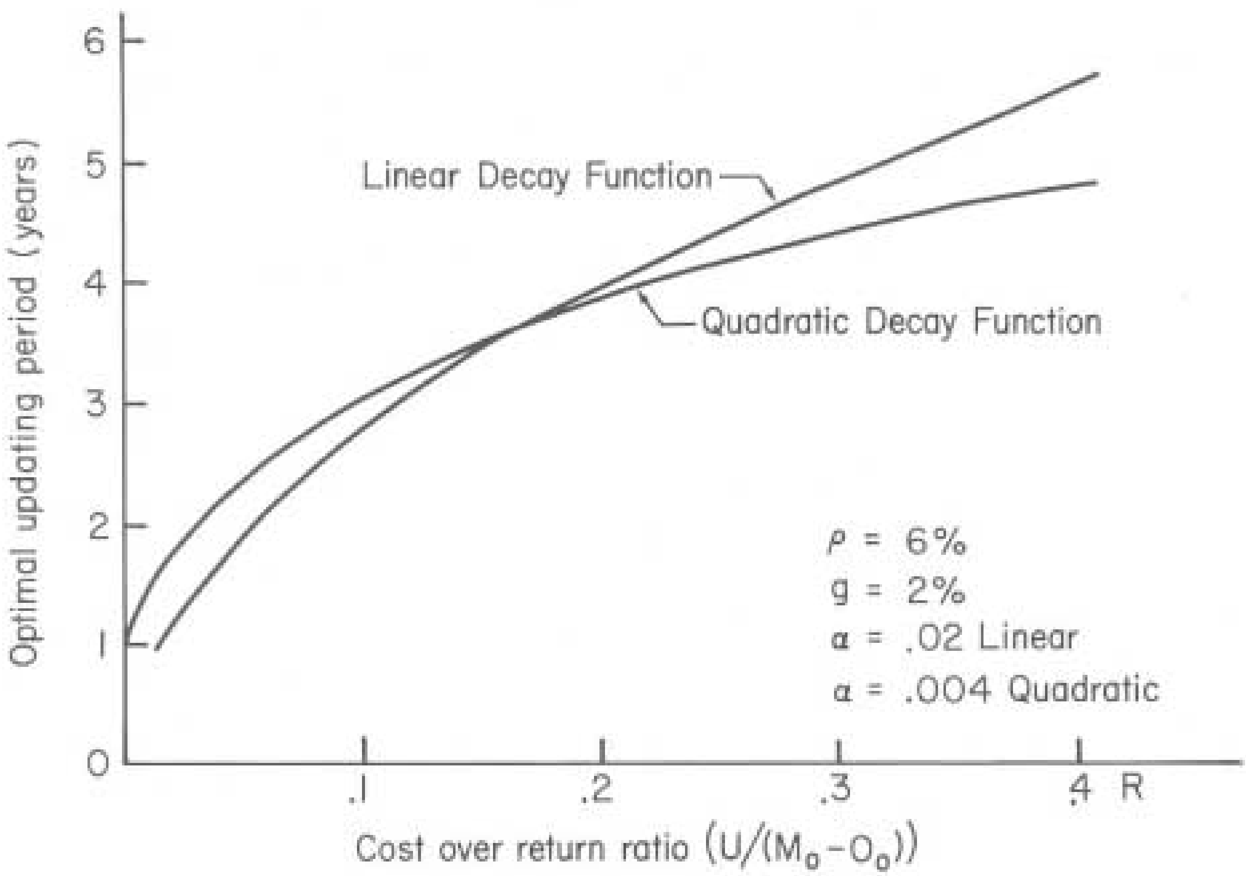

FIGURE II

COST OVER RETURN RATIO VS. OPTIMAL UPDATING SCHEDULE

With given prior probability of good and had customers, the decay of an existing system is relatively simple to measure. The f (t)(M_0 -0_0) for the j[th] period equals

$$\left[\left[\frac{N}{N_j}\right] M_j - O_o\right]$$ where N_J is the volume of applicants and M_j is the return

in the j[th] period. To establish the present value of the period's net return, the present value of all good and bad loans must be considered in the period when the loan is made. A loan made in period j that defaults in period (j+l) is counted with period j's returns. Fortunately, as most had customers default relatively early in the life of a loan, the system's effectiveness can be estimated fairly accurately before all loans are paid.

If the prior probability of good and bad customers change, the system's decay can still be estimated. Besides adjusting for the volume with the N/N_j term, $M_0$and 0_o will be different for an undated system. The estimate of the current system's decay requires an adjustment of the M_j value to give a proper comparison with the original M_0 value. M_0 is split into its components that are then adjusted for the prior probability shifts as shown in equation (9)

N

$$(9) \qquad \left[f(t)(M_o - O_o)\right]_j = \left[\frac{N}{N_j}\right]\left[\frac{P_{Go}}{P_{Gj}} R_j - \frac{(1-P_{Go})}{(1-P_{Gj})} L_j\right] - O_o .$$

P_{GO}/P_{GJ} is the ratio of the initial prior probability of a good customer to the j[th] period's revised priors. R_j is the present value of the after-tax contribution from credit extended in the j[th] period. And L_j is the net after-tax loss from bad customers extended credit in the j[th] period. Note that as more potentially good customers are rejected credit with decay, R_j is smaller. When a system decays, relatively more bad customers are extended credit. This is easily measured from actual results. However, if the proportion of applications rejected increases, are these good customers being rejected or has there been a shift in prior probabilities? The shift in priors discussed above could be merely an illusion resulting from the rejection of more good customers. This can be measured by assuming the system's rejection of good customers increases at the same rate as its acceptance of bad customers. For example, from a holdout sample a new system rejects 20 percent of known good customers and accepts 10 percent of known bad customers. With an initial prior probability of a good customer being 60 percent out of 100 applicants, 48 goods and 4 bads would be accepted while 12 goods and 36 bads would be rejected. If after some time the firm finds 6 percent of total applicants are bad, 40 percent are good, and 54 percent are rejected, the new prior probability of a good customer is approximately 54.2 percent while the system now accepts 13.1 percent of bad customers and rejects 26.2 percent of good customers.

When firms use a credit-scoring system merely as a guide in credit extension, the system's decay is somewhat more difficult to measure. Applicants must be treated as the system classifies them in order for the relative returns and losses to be determined. When the system says to reject an applicant and a loan is still made, the return or loss from this loan should not be considered in the system's return. When the system says to accept an applicant and a loan is not made, a subjective judgment is necessary. What percentage of these rejected customers would have been good customers? The percentage considered good is included in R_{gj}

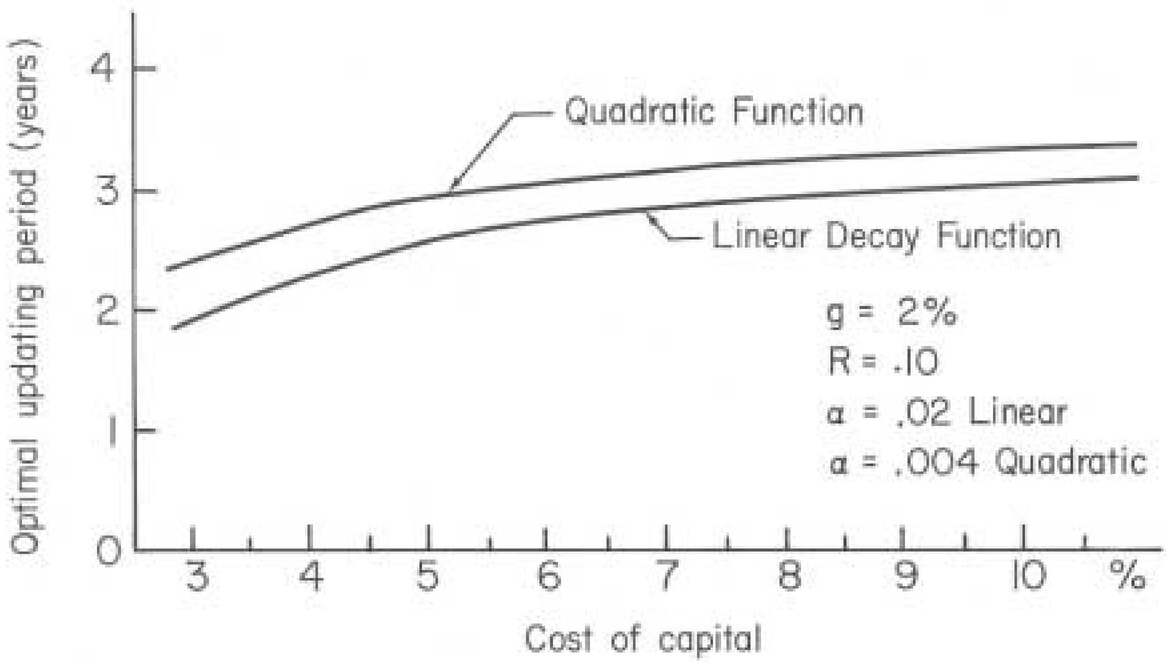

FIGURE III

COST OF CAPITAL VS. OPTIMAL UPDATING SCHEDULE

Once the decayed values over time are calculated, the correct decay function and its corresponding a value can be determined. Plotting the decayed results against time gives an indication of the functions to be considered. The α's are then estimated for these functions and the function having the that gives minimum variance estimate is selected. Any integrable, one-variable function can be considered.

An example can probably best show the process of selecting the appropriate function. The decay values by six-month intervals shown in Table 1 are plotted in Figure V. The a estimates are the minimum variance estimators under the conditions that when T=O, P=1.0. These are given in equations (10) and (11) where T is time and P is the performance ratio. These give α estimates of 0.02418 for the quadratic decay and 0.06088 for linear decay. The predicted decay for the two functions is also plotted in Figure V.

The error value (e) between the functions's predicted value and the actual value is found for each observation. Since its expected value does not equal zero with a forced intercept, the variance estimates are figured from ē. The variance estimate for the linear decay is 1.206×10^{-4} while the quadratic decay gives a variance estimate of 2.858×10^{-4}. Therefore, a linear decay function with a decay rate of 0.06088 is used to determine the updating period. The optimal updating period can now be determined for this problem using the methodology developed earlier.

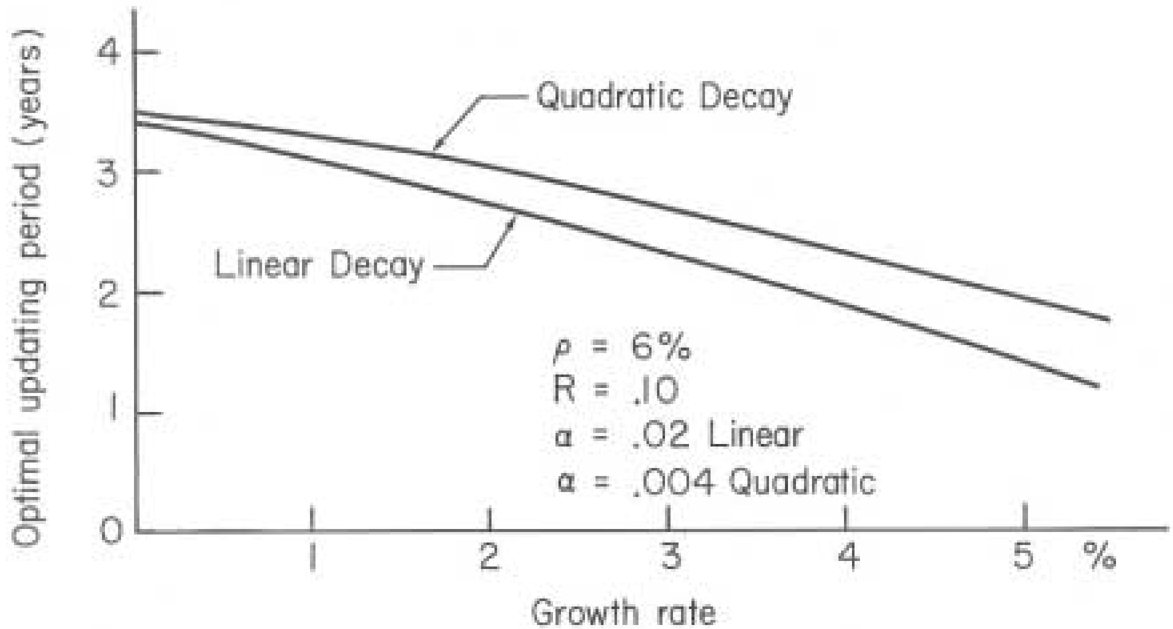

FIGURE IV

GROWTH RATE VS. OPTIMAL UPDATING SCHEDULE

TABLE 1
TIME VERSUS PERFORMANCE LEVEL

Time (T) in years	0.5	1.0	1.5	2.0	2.5	3.0
Performance $\frac{M_T - O_0}{M_O - O_O}$	0.97	0.95	.92	0.90	0.84	0.80

(10) $\alpha = \frac{\Sigma t - \Sigma etP}{Et^2}$ Linear Decay

(11) $\alpha = \frac{\Sigma t^2 - \Sigma Pt^2}{\Sigma t^4}$ Quadratic Decay.

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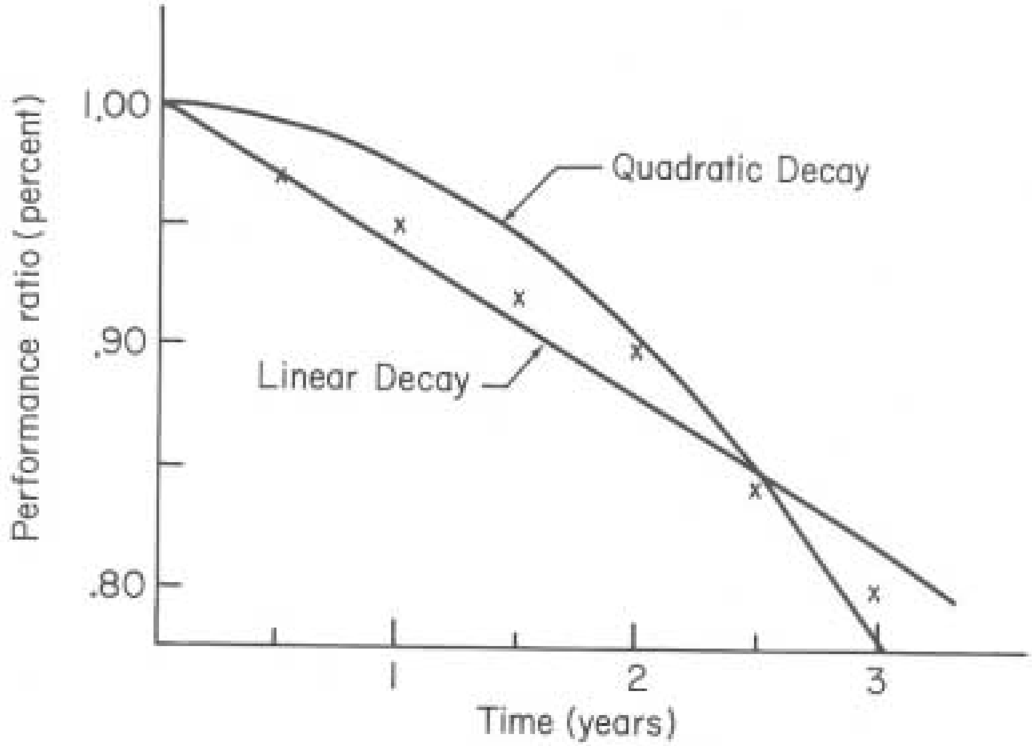

FIGURE V

DECAY TIME VS. PERFORMANCE RATIO

V. Conclusions

A creditor must have an overall system to determine the credit worthiness of applicants. Given the objectives of maximizing the value of a firm, a process to select the credit-screening system was developed that is general enough to accommodate varying loan profitabilities and losses and changing loan volumes.

An increasingly used screening system, credit-scoring, has a decaying performance level over time. To maintain an effective credit-scoring system, periodic revisions are required. The optimal updating cycle was developed to insure that the firm's wealth is maximized. The time between updatings was found to be directly related to the relative updating cost and the firm's discount rate and inversely related to the system's decay rate and the firm's growth rate. Except for the system decay, the management can directly determine the parameters. Measuring the system's decreased performance over time allows the decay function and rate to be estimated and the optimal updating cycle to be determined. By following these procedures, the creditor can select and maintain a credit-screening system that will maximize the firm's wealth.

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