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Problems in Applying Discriminant Analysis in Credit Scoring Models

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FOREWARD

Bob Eisenbeis first presented this paper on October 14, 1977, at a session of the Financial Management Association in Seattle sponsored by the Credit Research Center. While credit grantors are relying more heavily on credit scoring systems to make credit decisions, they must also assure themselves that their systems are in compliance with the Equal Credit Opportunity Act and the accompanying Regulation B. This paper makes an important contribution to understanding the problems involved in increasing the effectiveness of credit scoring while at the same time complying with the Federal requirements.

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Problems in Applying Discriminant Analysis in Credit Scoring Models

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Abstract

Since the mid- and late 1960s credit scoring and related loan review procedures have been applied with increasing frequency by financial institutions and other creditors. The passage of the Equal Credit Opportunity Act and promulgation of the Federal Reserve's Regulation B to implement this Act place an important responsibility on affected institutions employing screening models to ensure that their systems are statistically and methodologically sound. This paper reviews the types of credit scoring models that have been developed with particular emphasis on the methodological approaches that have been employed and the statistical problem associated with these models using discriminant analysis techniques. It is shown that in fact the statistical scoring models that have appeared in the literature have focused primarily on the minimization of default rates, which is only one dimension of the general credit-granting problem. Since profit maximization or cost minimization is, or should be, the objective of a scoring model, then most of the applied work seems incomplete. Ignoring these problems, it is also shown that the applied work typically suffers from statistical deficiencies. Some of the problems seem inherent in the discriminant analysis techniques employed or are not subject to easy remedy Oven the state of the art concerning estimation and sampling procedures.

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I. Introduction

Since the mid- and late 1960s credit scoring and related loan review procedures have been utilized with increasing frequency by financial institutions and other creditors.[1] Chandler and Coffman (1977), for example, report that credit-scoring systems are in wide use today. However, given the proprietary nature of these systems, precious little is known about the specific content of the models. There have been several credit scoring systems constructed by academics that have appeared in the journals. If these are representative of the types of systems actually being employed in industry, it would appear that a number of these systems could be expected to suffer from methodological and statistical problems that may have significant implications for the hundreds employing the models.

As Chandler and Coffman (1977) point out, these problems take on increasing importance given (1) the passage of the Equal Credit Opportunity Act (ECOA) in 1974 which prohibited discrimination in the granting of credit on the basis of sex or marital status and the amendments on March 1976 that added seven other prohibited factors including race, color, religion, national origin, age, receipt of public assistance benefits, and the good faith exercise of rights under the Consumer Credit Protection Act; and (2) the implementation of the Act through the Federal Reserve Regulation B. Particularly relevant to the consumer credit area are the provisions under the Act concerning the use of age in scoring models and the related interpretation under Regulation B describing the criteria scoring systems must satisfy with respect to sampling and validation before they would qualify to employ the prohibited age variable. These laws and new regulations place an important responsibility on any institution employing screening models to ensure that their systems are statistically and methodologically sound.

The purpose of this paper is to review the types of credit scoring models that have been developed. Particular emphasis in the next section is placed on the methodological approaches to selecting and developing screening models. The third section briefly discusses the various types of scoring models that have been developed. The fourth section explores the statistical problems associated with those models that have employed discriminant analysis techniques. The last section is a summary and conclusion.

II. Credit Scoring Model Methodology

Although there have been a number of attempts to derive credit scoring models, relatively few researchers have devoted specific attention to the underlying objectives one is trying to achieve in formulating or selecting among the alternative models that might be used. Notable exceptions have been Mehta (1968), Mehta (1970), Bierman and Hausman (1970), Dirickx and Wakeman (1976), Greer (1967a), Greer (1967b), Edmister and Schlarbaum (1974), and Long (1976).

One of the first to examine the problem of formulating an optimal credit granting policy was Greer (1967a) who proposed two models. One was designed to determine the optimal number of credit applicants to accept by maximizing "credit-related profits" which were a function of the present value of the profits from the current period credit sales, the present value of future profits from applicants granted credit in the current period, and the present value of profits from cash sales in both the current and future period. The second model also determined the optimal number of loans to market but included consideration of the opportunity costs of not granting loans to all applicants. Since this second model is formulated to differ from the first by only a constant, it is not surprising that Greer finds that the optimal number of loans to grant is the same with either

[1] A large number of nontechnical pieces have appeared discussing the use of quantitative credit scoring models. See for example Abate (1969), Amstutz and Liebman (1966), Biborosch (1965), Biborosch (1967), Cannellys (1967), Coakley (1970), Gooch, Wagner and Stratham (1973), Hammer and Orgler (1969), Harter (1974), McGrath (1960), Roy and Lewis (1970), Roy and Sanderson (1972), Weingartner (1966), Work (1967).

model. It is important to note that this model is in fact a multi-period model in the sense that the decision on the optimal number of loans to grant in a period is dependent upon not only the future value of the loans granted but also the future value and additional business that might be gained as a result of granting a given number of loans. As formulated the model does, however, have several limitations. First, it is based upon the firm's historical policies and portfolio experience to the extent that it relies on expressions for average default rates, costs of granting credit collections, and purchasing patterns of past customers. As such the proposed optimal policies aid only in deciding at what level to operate a given system. The model does not aid in actually making an individual credit decision, nor is it particularly useful for evaluating alternative systems not yet in operation, at least in its present form.

Shortly following Greer's work, Mehta (1968), (1970) presented a series of sequential decision models using different and successively more costly bits of information to refine the credit granting process. Mehta's (1968) approach to the credit granting process was quite straightforward and based upon two simple assumptions. First, he assumes that an individual's past credit performance serves as a useful guide to project future credit performance. Second, he argues that information is costly and not all information may be needed to make a decision on every applicant. A creditor is thus faced with the problem of deciding how much information to collect before making the credit decision. Three costs become relevant:[2]

acceptance cost = bad debt cost + investment cost + collection cost

rejection cost = lost sales cost

cost of further information = cost of acquiring additional information + cost of additional decision.

The creditor then is to go through a sequential process attempting, based upon past experience, to select the alternative that minimizes costs. Each time the alternative to seek more information is selected, more costly information is assumed to be acquired.

In his second paper Mehta (1970) extends the earlier static model to take into account optimal decisions for collection, which were previously assumed to be exogenous, and to recognize the value of future sales and credit requests that might result. A Markov process is used to estimate the values of the additional revenues and costs of collecting accounts i periods old.

Bierman and Hausman (1970) focus on the class of models involving multiple requests for single period loans. In each case it is assumed that a loan is granted and is either repaid in full or not collected and written off during the period requested. At the end of each period a new loan may be requested, and so on for n periods. The essential features of the models are that they allow for consideration of estimates of prior probabilities of collection, consideration or valuation of the customer relationship over n periods, revision of the estimates of the probabilities of repayment based upon previous periods' payment performance, incorporation of discounting in valuing future period receipts and collection costs, consideration of the probability that a customer may terminate the customer relationship at some future date, and allowance for the fact that the amount of credit granted may affect repayment performance. The models are formulated as dynamic programming models in which the revisions in the estimates of collection probabilities are assumed to follow a Beta distribution with parameters n and r denoting n periods and r collections. Dirickx and Wakeman (1976) extended the Bierman-Hausman models to allow for collection after the end of the first period and to allow for partial repayments. These models are still limited, however, by representing a loan as essentially a single period instrument. As such they are not particularly applicable to the installment lending area.

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[2] bad debt cost = (probability of nonpayment)-(variable product cost)
lost sales cost = (probability of payment)-(contribution margin)

Edmister and Schlarbaum (1974) directly address the problems of selection and evaluation of alternative credit granting systems. Under the assumption that management's objective is to maximize the market value of shareholders' equity, they argue that the goal of any credit analysis system should be to maximize the "expected net present value of granting loans to applicants." They proceed to construct an objective or criterion function that is shown to be dependent on (1) a credit system's acceptance rate, (2) the expected returns and losses on good and bad loans that have been accepted, and (3) the cost of credit analysis and processing. In addition they recognize that certain dimensions of an institution's credit policies, such as interest rates and collection policies, will affect not only the number of applications received but also the net present values of good and bad loans. They do not deal with these endogenous components to evaluating credit systems and proceed to illustrate the selection among alternative systems with given net present values and number of applications. As such, their work is a partial equilibrium approach and nonoptimal. Omission of consideration of the endogenous components eliminates the most interesting and relevant behavioral considerations in the loan-granting problem.

Long (1976) extends the Edmister and Schlarbaum approach to explicitly recognize (1) startup and updating costs, (2) variation in system efficiency over time, and (3) changes in the number and quality of loan applicants. He then proceeds to investigate the effects of alternative assumptions about system decay on update policies.

From a review of the various approaches that have been taken to formulate credit model selection objectives it would appear that there are several key elements to the credit granting decision that have important implications for the construction of a credit scoring model.

First, the credit granting decision is in fact a multi-period problem that, depending upon the type of credit involved, may have two dimensions. In the context of the Bierman-Hausman type model, the granting of credit in one period is but part of a customer relationship which extends over many periods. The decision to grant credit affects the value of that customer relationship both over the period of a particular loan and also over the life of the customer relationship. Clearly such a view of the credit process is particularly applicable to the business loan activities of a financial institution and has in fact been modeled by Hester and Pierce (1975) and by Hester (1962). It is not, however, without relevance to the consumer lending area where it is quite clear that the provision of credit is tied or linked to the provision of other financial and nonfinancial services by creditors over periods greater than the life of a loan. It is also clear from the approaches of many of the authors, particularly Edmister and Schlarbaum (1974) and Long (1976), that a credit decision is really a multi-period decision over which a loan generates a flow of revenues until it either is paid off in full or defaults, after which a portion of the value of the principle may be recovered.[3] Thus, the value of a loan is determined not only by whether it is paid in full' but also in the case of default, by the length of time it is current, collection costs, and the realizable value of any collateral that may remain. In such a framework it could be that, even if it were 100 percent certain that a loan might not be paid in full, it may still be worthwhile granting the loan if it is current for a sufficient length of time. This is particularly true if collection costs are low, and the collateral has sufficient recovery value. The possibility that a defaulted loan may still be profitable has been given little attention in the published work to date.

Second, Mehta (1968), (1970), Bierman and Hausman (1970) and Greer (1967a), (1967b) make the point that there is a need to consider the cost of information in formulating a credit-granting scheme. Because information on past credit performance, while not costless, is relevant to assessing future credit performance, there is a need to weigh the value of collecting additional information against costs before deciding when is the appropriate time to make the credit decision.[4]

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[3] Dirickx and Wakeman (1976) generalize the Bierman-Hausman (1970) model to include partial and/or full pay back over more than one period.

[4] Both Greer (1967b) and Chandler and Ewert (1975) address this problem directly.

Third, most of the credit policy models discussed explicitly or implicitly assume that the prospective lender judges the riskiness of a borrower in terms of either default probabilities or opportunity costs in deciding whether or not to grant a loan. Greer (1967b) assumes such a ranking. Bierman and Hausman (1970) and Dirickx and Wakeman (1976) employ a Bayesian approach to estimating default probabilities. Edmister and Schlarbaum (1974), Long (1976) and Mehta (1968)rely on average historical loss experience for given categories of loans.

Finally, in multiperiod/multidecision models, such as the customer relationship approach of Bierman and Hausman (1970) or Mehta (1970), there may be a need to readjust estimates of expected default probabilities over time based upon customer performance.

Having reviewed the general approaches to the credit granting problem and examined what appear to be the key features that a credit scoring model might be required to possess, we now turn to a brief review of the types of models that have been proposed in the literature to see how well they measure up.

III. Review of Credit Scoring Models

In general, the credit scoring models developed have focused on two categories of loans: (1) consumer loans including installment type lending and credit cards[5] and (2) commercial loans including term loans, regular commercial and industrial loans, and loans to minorities and small businesses.[6] In addition, the models have been developed for application to two district phases of the lending function. Most of the models have focused on credit granting decision, but at least three studies have dealt with the loan servicing and review functions.[7]

The typical approach in the models noted above (supra, notes 5 and 6) is to categorize sample loans into two mutually exclusive groups: "good loans," which are those that will be paid or are current; and "bad loans," which are slow paying, delinquent or in default.[8] The bad loans are viewed as being risky while the good loans are not. Usually a discriminant function or related procedure is estimated from the pool of loans that had already been granted. A classification rule is then formulated that is designed to distinguish, or discriminate, between the groups of good and bad loans, while minimizing the overall error rates or costs of misclassification.

It is interesting to note how few of the models actually satisfy the criteria noted in Section II. For example, none of the models were specified so as to be a multi-period model, to take into account the value of the customer relationship, or to recognize that a defaulted loan may still be profitable-albeit more risky--under certain circumstances. On this latter point, the critical analytical question is not simply whether or not a loan will go into default. Rather, the problem centers on assessment of the probability of default in periods 1, 2,..., t.[9]

The only area when the reported models (supra, notes 5 and 6) dealt with the criteria in Section II was in the area of estimation of default probabilities. Even here there were clear divisions among the models. Those of

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[5] Work on consumer lending models includes Apilado, Warner, and Dauten (1974), Boggess (1967), Chatterjee and Barcun (1970), Durand (1941), Edmister and Schlarbaum (1974), Ewert and Chandler (1974), Hettenhouse and Wentworth (1971), Lane (1972), Meyers and Forgy (1963), Smith (1964).

[6] Work on commercial loans includes, Altman, Margaine, Schlosser and Vernimmen (1974), Bates (1973), Chesser (1974), Cohen, Gilman and Singer (1966),Edelstein (1975), Edmister (1971), Ewert (1969), Orgler (1970), Pratt and McGhee (1967).

[7] Ewert and Chandler (1974), Chesser (1974), and Orgler (1971).

[8] For example, Bates (1973), classified loans more than 60 days past-due as "bad" and current loans as "good." Lane (1973) investigated three groups--debt counselors, bankruptcies, and bankruptcies filed under Chapter III. Edelstein (1975) defined good loans as all those whose payments were on schedule and all others were considered to be bad.

[9] The models of Mehta (1968), (1970) and Bierman and Hausman (1970) suggest that the probability of default in period t may not be independent of payment performance in periods t-1, t-2,..., etc.

Lane (1972), Chatterjee and Barcun (1970)[10] and Durand (1941) specifically had as an objective to minimize the probabilities of misclassification. Most of the remaining scholars are silent about the objectives of their models, although it is probably fair to conclude that they are implicitly attempting to minimize classification errors.

With few exceptions, then, the models discussed in this section are essentially directed toward only one dimension of the credit granting function, albeit a critical one, and that is the assessment of the likelihood that loans will default, go "bad," or experience difficulty over their life. In this sense they are all single-period models with that period being the life of the loan. No attempt is made to determine at what specific intervals over the life of a loan it is more or less likely to default or become slow paying.[11] Within the scope of this narrower objective, we now turn attention to how well it is executed.

IV. Statistical Problems in Credit Scoring Models

Most of the models reviewed in the previous section suffer from statistical problems that may affect the reliability of the estimates of default probabilities. Particularly for the consumer credit granting models, it is important to understand the dimensions of these statistical problems, since as Chandler and Coffman (1977) note, only if credit scoring model is "demonstrably and statistically sound" can it satisfy the regulatory standards under the Federal Reserves Regulation B. Moreover, to the extent that a model is subject to criticism on statistical grounds, vulnerability under the "effects test" is likely to be increased.[12]

Since all but a few of the models that have appeared in the literature have employed discriminant analysis procedures, attention is focused on the problems that can be identified with the use of this technique.[13]

As has been noted elsewhere by Eisenbeis (1977), the statistical problems in the applied discriminant analysis literature may be categorized into seven different types.[14] These are

(1) violation of the assumption about the underlying distributions of the variables

(2) use of linear discriminant functions instead of quadratic functions when the group dispersions are unequal

(3) improper interpretation of the role of individual variables in the analysis

(4) reductions in dimensionality

(5) problems in the definition of the groups

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[10] Chatterjee and Barcun (1970) did consider costs but only as a device to show how the cut off points and error rates were shifted and not as a part of the model. Durand (1941) also considered costs but not as part of his empirical model.

[11] An exception is the model of Ewert and Chandler (1974) which focused on the likelihood that once a loan becomes slow paying in one period, it would continue to be slow paying in subsequent periods. This model is not, however, a credit granting model; it is a loan review and internal control model. In this respect, it is more closely linked to the work on optimal foreclosure policies of Pye and Tezel (1974) and Mitchner and Peterson (1957).

[12] See Chandler and Coffman (1977) for a discussion of the "effects test."

[13] The models of Chatterjee and Barcun (1970), Chesser (1974), Cohen, Gilman, and Singer (1966), Edelstein (1975) and Smith (1964) employ other statistical methods. For a general discussion of the theory of discriminant analysis see Eisenbeis and Avery (1972), Lachenbruch (1975), or Cooley and Lohnes (1971), (1962).

[14] Each of these issues are explored in detail in Eisenbeis (1977) with numerous references to the relevant theoretical and applied literature.

(6) use of inappropriate a priori probabilities and/or costs of misclassification

(7) problems in the estimation of classification error rates to assess the performance of the model.

A review of the credit scoring applications suggest that additional problems are associated with selection of the analysis samples.

Each of these problem areas will be briefly discussed in turn.

Distributions of the Variables

One of the critical assumptions in discriminant analysis is that the variables describing the members of the groups being evaluated are multivariate normally distributed. None of the models referenced in this paper devote any attention to this problem. Clearly, those models employing categorical variables violate the distributional assumptions,[15] and hence, may be immediately open to challenge. A successful defense would have to rest on a discussion of the robustness of the procedures employed to differing distributional assumptions.[16] Alternatively, models described by Hills (1967), Linhart (1959) and Chang and Afifi (1974) employing nearest neighbor techniques or methods of subdividing the populations might be used.

Equal vs. Unequal Dispersions

The decision to employ linear as opposed to quadratic discriminant analysis techniques theoretically rests upon tests for the equality of the group dispersions. When the dispersions are equal, linear procedures should be employed, and when the dispersions are unequal, quadratic procedures are appropriate.[17] Of the credit scoring models reviewed, only Bates (1973) Lane (1972), Durand (1941), and Ewert and Chandler (1974) devoted any attention to the appropriate functional form of the model. In this connection, it should also be noted that it is easy to show that use of dummy variables in a discriminant analysis model implies not only that the distributional assumptions are violated, but also that a quadratic model should be used.

The Role of Individual Variables

As Chandler and Coffman (1977) point out, there is a high probability that court tests of the validity of credit scoring models may constrain the variables included in a model requiring the user to demonstrate that each variable contributes significantly to the overall discriminatory power of the model. This possibility poses a serious potential problem for users of discriminant analysis Procedures, since there are no statistical procedures analogous to those in regression analysis to test for the significance of individual coefficients. In the linear case, it is possible to test for the conditional significance of individual variables and presumably analogous tests could be formulated in the quadratic case. But as of yet these tests are not widely available in useable form.[18] The lack of exact tests and the need to rely on approximations would certainly complicate the defense of many discriminant analysis credit-scoring models in court.[19]

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[15] See Eisenbeis (1977) for a discussion and references.

[16] All of the variables in Edmister (1971) are categorical and all but one in Orgler (1970) are dummy variables.

[17] Monte Carlo studies by Marks and Dunn (1974) suggest that there may be efficiency trade-offs between use of linear and quadratic rules and sample sizes. When samples are small and the number of variables relatively large, linear rules may give more efficient estimates of the expected error rates than quadratic rules even when the population dispersions are unequal.

[18] Martell (1977) employs a test proposed by James (1954) to test for the overall significance of the model when the group dispersions are unequal.

[19] At least six alternative methods have been proposed and employed, see Altman and Eisenbeis (1976).

Reduction in Dimensionality

The problems associated with reducing the number of variables in a model pose no special problems for credit scoring models aside from those generally noted in Eisenbeis (1977).

Problems in the Definition of the Groups

Discriminant analysis assumes that the groups being investigated are discrete and identifiable. As noted previously, in most of the applications this has been interpreted as categorizing loans for the purposes of estimating the scoring models as either "good," in which case they were either current or had been paid off, or as "bad," in which case they were in default or in some way delinquent. For the purposes of constructing the models, these groupings appear to satisfy the discriminant analysis assumptions, and pose little difficulties so long as one is willing to regard risk as a discrete rather than a continuous concept and to disregard recovery values or collateral as a means of reducing creditor risk.

Use of Inappropriate a priori Probabilities and Costs of Misclassification

The standard discriminant analysis classification rules--both linear and quadratic--incorporate a priori probabilities to account for the fact that the populations or groups being investigated occur with different relative frequencies in the population. Furthermore, it is also possible to adjust for the fact that one type of misclassification may even be more costly than another.[20] Relatively little attention has been paid to the selection of the appropriate a priori probabilities and most have simply assumed that both the a priori probabilities and costs of errors are equal.[21] It can be demonstrated that proper choice of the a priori probabilities is critical to assessing the performance of the model and estimates of the default rates. Use of a priori probabilities different from the population priors will result in estimates that may bear little relationship to what might be expected when the model is applied to the population or what the true error rates should be. Moreover, it will not result in minimization of the probabilities of misclassification. In the models reviewed in this paper, most simply assumed equal a priori probabilities or did not even mention what priors were employed.

Estimation of Classification Error Rates

Particularly critical under the ECOA will be the ability to illustrate accurately or to estimate the overall performance of the models in terms of the objective function employed. For those attempting to minimize classification error rates, the researchers will be forced to trade off the costs of sample size and of expansion of the variance of error estimates. It has been well publicized that employing the samples used to construct the classification rules to estimate expected error rates leads to a biased and optimistic prediction of how well the model will perform. A number of alternative methods have been proposed, the most widely used and most expensive being the hold-out method. It is also interesting to note, however, that tests indicate that the hold-out method is not clearly superior to these alternatives.[22] Of the applied work Orgler (1970), Ewert (1969), Ewert and Chandler (1974), Apilado, Vincent and Dauten (1974), Myers and Forgy (1973), employ the hold-out method. Hettenhouse and Wentworth (1971), and Bates (1973) simply allow for user selection of desired error tradeoffs, presumably based upon reclassification of the original samples. Lane (1972), Orgler (1971), and

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[20] The costs of errors are fixed in that it is assumed that the cost (for example) of a default is invariant with the characteristics of the transaction. That is, it assumes that losses or costs are the same for all errors of the same type.

[21] Only Pratt and McGhee (1967) and Hettenhouse and Wentworth (1971) consider costs at all in those models employing discriminant analysis. Chatterjee and Barcun (1970) consider costs in their nearest neighbor scheme.

[22] For a discussion of the methods see Eisenbeis and Avery (1972) and also Eisenbeis (1977).

Altman, Margaine, Schlosser, and Vernimmen (1974) employ the original sample method, and Edmister (1971) used a synthetic method.

Selection of Analysis Samples

As Chandler (1977) notes, a special problem arises in credit scoring models because they are usually developed from samples of loans that have been granted rather than based upon lending experience with the population of potential borrowers or through-the-door applicants. Short of granting loans to all comers, the potential user of a credit-scoring model is usually forced to estimate a model with the truncated population of approved borrowers. Such a method is, however, subject to severe problems. Avery (1977) has investigated the effects on a scoring model, with known population parameters, of estimating the model with truncated-normal samples. His results are quite alarming. First, he notes, that even when the underlying populations have equal dispersions, use of truncated samples will lead one to reject the equal dispersion hypothesis and employ quadratic rather than linear rules. Furthermore, one obtains biased estimates of the group means and dispersions, the true cut-off point, and true error rates. Furthermore, the direction and extent of the bias is shown to depend upon the original truncation points and hence is not usually known ex-post. In most instances then, without knowing the truncation system, it is not possible to estimate in which direction the bias lies. The results of Avery's work are of concern for two reasons. First, systems estimated from loans that have been granted are clearly biased, and the direction and size of that bias is not known. Second, even if a lender decided to incur the costs of granting loans to all through-the-door applicants, there is still no assurance that such a sample would be representative of the population of potential borrowers. If borrowers are aware of an institution's credit policies, there is a self-prescreening that can take place because of the autogressive nature of the process so that even the resulting through-the-door samples might be biased.

Chandler and Coffman (1977) discuss several alternatives to this problem of validation including an interesting heuristic method based upon sampling theory. Shinkel (1977) investigates two alternatives. One, employed by Long (1973), uses both "good" and "bad" accepted applicants together with the rejected applicants to estimate, based upon reclassification of the applicants, what proportion of the rejects are likely to be "good" and "bad." The second is a method called "augmentation" developed by Fair, Issac and Company to weight the good and bad applicants in the accepted group by an estimate of the likelihood that each would be accepted. Shinkel (1977) concludes that both methods also yield biased estimates of the predictive ability of the scoring model. Avery (1977) has proposed a maximum likelihood method to correct for the bias in the estimates of the means and dispersions to construct consistent estimates of these parameters which can be used to construct unbiased estimates of the error rates. This area is, however, as yet anew one for investigation and promises to be important in view of ECOA.

V. Summary and Conclusion

This paper has reviewed the methodological foundations for the formulation and selection of credit scoring models in order to place the applied literature in perspective. It was shown that in fact the scoring models that have appeared have focused primarily on the minimization of default rates which is, in fact, only one dimension of the more general credit-granting problem. To the extent that profit maximization or cost minimization is, or should be, the objective of a scoring model, then most of the applied work seems incomplete. Even ignoring these problems, it is also shown that the applied work typically suffers from statistical deficiencies. Of equal concern is that some of the problems seem inherent in the discriminate analysis techniques employed or are not subject to easy remedy given the state of the art concerning estimation and sampling procedures.

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